Consumption dynamics and the relative price of consumer durables*

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PRELIMINARY AND INCOMPLETE

February 15, 2019

Extended Abstract

We explore the business-cycle implications of investment-specific technology for consumer durables. Standard models suggest that shocks to the relative price of consumer durables imply a negative correlation between the relative price and the quantity of consumer durables. We look into the detailed personal consumption expenditure category and document that the correlation between the cyclical components of the relative price and quantity is significantly negative for a subset of consumer durables which also experienced a large decline in their relative prices. Moreover, the relative price of these durables are also more volatile than that of other durables, and the composition of these durables as a share of total durables is rising. To explore the quantitative importance of relative price shocks, we estimate a model with durable investment-specific technology. When using only aggregate consumer durables as an observable, we find that investment-specific technology does not

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*We thank seminar participants at Seoul National University, Tsinghua University, and Korea Development Institute for precious comments. All errors are ours.

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matter for business-cycle fluctuations. We argue that this is due to the attenuated correlation between price and quantity for aggregate consumer durables. We plan to estimate the model with two types of consumer durables as observables, to ask whether the importance of durable investment-specific technology can be restored when heterogeneity in the technology of consumer durables is considered.

Keywords: Investment; Durables; Investment-specific productivity.
JEL Classification Numbers: E22, E32, O22.

1 Introduction

Studies on business cycles have emphasized investment behavior as key to understanding the sources of economic fluctuations. In the recent literature, a variety of investment shocks are proposed and estimated as driving forces of business cycles. Influential papers find that shocks to investment demand matters in accounting for the dynamics of investment and output, while the role of investment-specific technology is more controversial, especially when the observed relative price of investment series is used in the estimation (Justiniano, Primiceri, and Tambalotti, 2010, 2011).[1]

In this paper, we explore the business-cycle implications of investment-specific technology by investigating household investment of consumer durables. While data on consumer durables are widely used in the business-cycle literature, they are frequently combined with the business investment series to represent the total productive capital investment in the economy. In particular, the household decision on consumer durables is typically not modeled in business-cycle models that study investment shocks.

In this paper, we find that separating household durable expenditures from capital investment in both the model and the data provides us with a new source of information that can be used to quantify the importance of investment prices on business cycles. Moreover, we document that the price and quantity of both technology-embedded household durables and classical household durables exhibit markedly different cyclical properties. By allowing for two types of household durables and their prices in a business-cycle model, we re-estimate the importance of investment shocks and reach a strikingly different conclusion compared to the literature, in that we find that the observed investment price shocks for technology-embedded households goods are a leading source of economic fluctuations.

[1]By investment demand shocks, we include the marginal efficiency of investment as defined in Justiniano, Primiceri, and Tambalotti (2010). They find that this process can proxy for credit spreads observed in the data.

Related literature. Seminal papers by Greenwood, Hercowitz, and Krusell (1997, 2000) and Cummins and Violante (2002) argue that productivity specific to the investment sector can account for the majority of long-run growth, and is also important in accounting for business cycles. Fisher (2006) estimates a structural vector autoregression model and find that investment-specific technology shocks are key to output and investment dynamics. Nevertheless, estimations in dynamic, stochastic, general equilibrium models find limited importance of these shocks, when non-technology-related investment shocks are also considered (Justiniano, Primiceri, and Tambalotti, 2010, 2011). Schmitt-Grohe and Uribe (2011) argue that common components to neutral and investment-specific technology could account for output dynamics. In this paper, we revisit the cyclical relevance of investment-specific technology shocks by looking into the details of consumer durables.

2 Simple model of consumer durables

In this section, we study the relationship between the relative price of durables and consumption dynamics through a simple business-cycle model of consumer durables. In particular, we study to what extent the correlations between the relative price of durables and the quantities of both durable investment and nondurable expenditure could be informative in understanding the importance of investment-specific technology for consumer durables.

2.1 Households

An economy is populated by a large number of identical households with preferences described over nondurable consumption C_t, stock of durables D_t, and hours worked h_t,

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t [\log C_t + \psi_D \log D_t - \psi_h h_t^2/2], \tag{1}$$

where $\beta < 1$ denotes the household subjective discount factor.

The household sequential budget constraint is

$$C_t + p_t^d N_t = w_t h_t + \Pi_t,$$

where p_t^d is the relative price of consumer durable goods. The physical units of durable investment is denoted as N_t. Real wage is w_t and the real profit of the firm is Π_t since households are the owners of firms.

Household durable stock D_t evolves over time according to

$$D_t = (1 - \delta_d) D_{t-1} + \mu_t N_t, \tag{2}$$

where μ_t is the marginal efficiency of durable investment.

2.2 Firms

The production function of the total output firm is given by

$$\tilde{Y}_t = F(h_t), \tag{3}$$

where $\tilde{Y}_t$ is total production in nondurable consumption units and h_t is the labor input.

The total output producing firm maximizes the following profit function:

$$\Pi_t = \tilde{Y}_t - w_t h_t,$$

by choosing labor input in a competitive market.

Durable investment goods are produced by a durable-specific production function:

$$N_t = z_t^d \tilde{N}_t,$$

where N_t is the consumer durable investment good, $\tilde{N}_t$ is the input, and z_t^n is the durable-specific technology.

Producer of the durable investment good maximizes the following profit function:

$$\Pi_t^d = p_t^d N_t - \tilde{N}_t,$$

by choosing the level of input for durable good production in a competitive market.

2.3 Market clearing and exogenous processes

Total output is used either for nondurable consumption, or for input in the production of durable goods. The goods market of the economy clears:

$$\tilde{Y}_t = C_t + \tilde{N}_t. \tag{4}$$

The exogenous processes for durable-specific productivity is

$$\ln z_t^d = \rho_z^d \ln z_{t-1}^d + \sigma_z^d \varepsilon_{z,t}^d. \tag{5}$$

Lastly, the exogenous processes for the marginal efficiency of durables is

$$\ln \mu_t = \rho_\mu \ln \mu_{t-1} + \sigma_\mu \varepsilon_{\mu,t}. \tag{6}$$

2.4 Equilibrium

The equilibrium of the model can be summarized by five endogenous variables $\{C_t, D_t, N_t, h_t, p_t^d\}$ that satisfy the following equations:

$$\psi_h C_t h_t = F_h(h_t),$$

$$
\begin{aligned}
\frac{\psi_D}{D_t} &= \frac{p_t^d}{C_t} - \beta(1-\delta_d)\mathbb{E}_t \frac{p_{t+1}^d}{C_{t+1}}, \\
p_t^d &= \frac{1}{z_t^d}, \\
C_t + p_t^d N_t &= F(h_t), \\
D_t &= (1-\delta_d)D_{t-1} + \mu_t N_t,
\end{aligned}
$$

where $\{z_t^d, \mu_t\}$ are the two exogenous variables that follow equations (5) and (6).

2.5 Relative price of durables and consumption

It can be shown that the log-linearized equilibrium conditions around the non-stochastic steady state are

$$
(1-m)\hat{p}_t^d = -\left(\frac{\alpha}{1+\alpha} + m\right)\hat{c}_t - (1-m)\hat{n}_t, \tag{7}
$$

$$
\hat{d}_t = (1-\delta_d)\hat{d}_{t-1} + \delta_d(\hat{\mu}_t + \hat{n}_t), \tag{8}
$$

$$
-(1-\beta(1-\delta_d))\hat{d}_t = (1-\rho_z^d\beta(1-\delta_d))\hat{p}_t^d - \hat{c}_t + \beta(1-\delta_d)\mathbb{E}_t\hat{c}_{t+1}, \tag{9}
$$

where $m = C_{ss}/\tilde{Y}_{ss}$ is the steady state ratio of nondurable consumption to total output, and $F(h) = h^{1-\alpha}$.

Starting from the steady state, we study the initial response of the economy to a shock to the relative price of durables. First, we start from the extreme example of $\delta_d = 1$. In this case, both consumption goods C_t and D_t are technically nondurable and the only difference between the two is their relative price. One can show that $\hat{n}_t = -\hat{p}_t^d$ and $\hat{c}_t = 0$.

Second, we study the general case of $\delta_d < 1$. Denote ρ_c as the endogenous persistence of future nondurable consumption conditional on a relative durable price shock (i.e. $\mathbb{E}_t\hat{c}_{t+1} = \rho_c\hat{c}_t$). Let $\bar{\rho}_c$ be

$$
\bar{\rho}_c \equiv \frac{1}{\beta(1-\delta_d)}\left[1 + \frac{\alpha/(1+\alpha)+m}{1-m}(1-\beta(1-\delta_d))\delta_d\right] > 1. \tag{10}
$$

The following proposition holds.

Proposition 1 (Relative price and consumption response) *Assume that equations* (7)–(9) *hold. At the steady state, a shift in the relative price of durables implies that on impact,*

1. *durable expenditures respond by*

$$\hat{n}_t = -A\hat{p}_t^d,$$

2. *nondurable expenditures respond by*

$$\hat{c}_t = \left(\frac{1-m}{\alpha/(1+\alpha)+m}\right)(A-1)\hat{p}_t^d.$$

The expression for A *is*

$$A = \frac{(\alpha/(1+\alpha)+m)(1-\rho_z^d\beta(1-\delta_d)) + (1-m)(1-\rho_c\beta(1-\delta_d))}{(\alpha/(1+\alpha)+m)(1-\beta(1-\delta_d))\delta_d + (1-m)(1-\rho_c\beta(1-\delta_d))}.$$

If $\delta_d = 1$*, then* $A = 1$*. On the other hand, if* $\delta_d < 1$ *and* $\rho_c < \bar{\rho}_c$ *where* $\bar{\rho}_c$ *is defined in* (10)*, then* $A > 1$*.*

Proof. *In the appendix.* ■

While the value of $\bar{\rho}_c$ should be verified numerically, since $\bar{\rho}_c > 1$, our restriction of $\rho_c < \bar{\rho}_c$ is quite general and includes the case where nondurable consumption is a martingale.

The first observation from this proposition (when $\rho_c < \bar{\rho}_c$) is that durability plays a key role in understanding the consumption response to the relative price movement. When $\delta_d = 1$, the durable good is technically nondurable. In this case, a shift in the relative price of durables has no implications for the other nondurable good. However, when $\delta_d < 1$, then $A > 1$ which implies that nondurable goods and the relative price of durables move in the same direction on impact. This is due to the accelerated demand channel for durables when the durable stock is larger than its flow.

The second observation is that consumption responses depend on the relative steady-state production between the two goods. Holding ρ_c constant, an increase in m implies an increase in A. That is, when the economy mostly produces nondurables relative to durables at the steady state, then a one percent fall in the relative price of durables leads to an even larger increase in durable expenditures.

The observations from our proposition are based on regularity conditions on ρ_c, which is a value that can only be determined numerically conditional on a shock. In the next section, we verify that the regularity conditions hold in our calibrated model and provide an intuitive explanation of our observations.

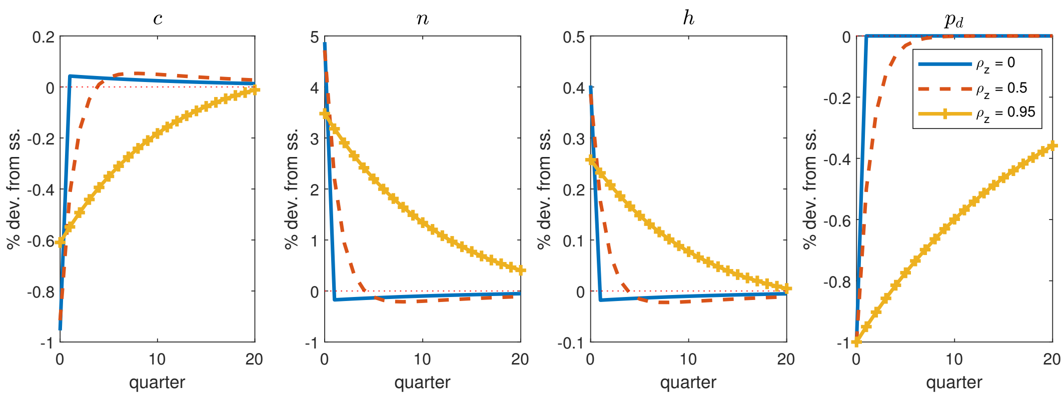
2.6 Impulse responses and discussion

To verify the proposition above, we plot the impulse response functions for the model, by setting $\alpha = 0.37$, $\beta = 0.99$, $\delta_d = 0.025$, and $h_{ss} = 0.3$. In panel (a) of Figure 1, we find that with regards to a fall in the relative price of durables, durable investment increases and nondurable consumption falls. Total hours increase. We observe that the qualitative movements are robust to the persistence of the durable investment-specific technology. However, in terms of the impact movements, lower persistence leads to quantitatively stronger initial responses of both durables and nondurables. In panel (b), we hold $\rho_z = 0.95$ and experiment with three different values for m. We find that with an increase in m, durable investment increases but the fall in nondurable consumption is also smaller. The increase in total hours is smaller with higher m.

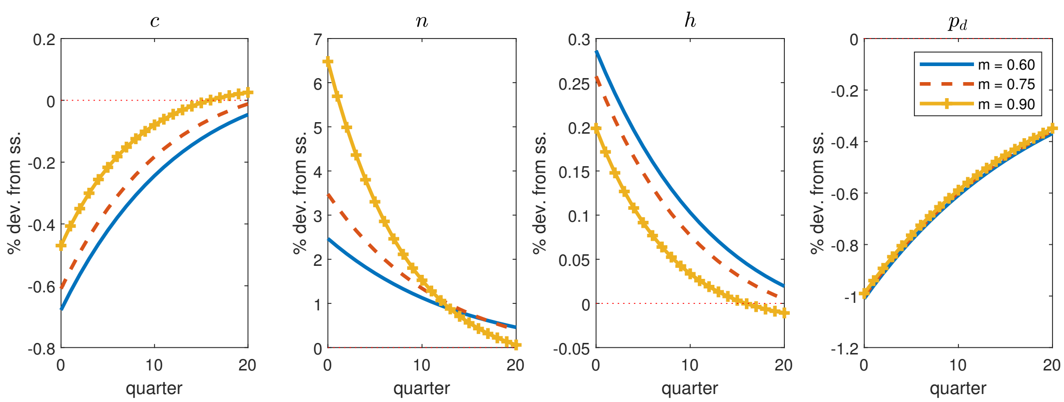
The impulse responses numerically verify the above proposition. To provide an intuitive explanation, notice that there are two channels through which the relative price of durables affect nondurable consumption. First is the substitution effect due to changes in relative prices. When the relative price of

Figure 1: Impulse response functions

(a) Relative price persistence



(b) Steady state consumption share of nondurables



Note: Impulse response functions. The first panel sets $m = 0.75$ and varies the persistence ρ_z of the durable investment-specific technology process. The second panel sets $\rho_z = 0.95$ and varies m.

durables declines, nondurables become more expensive than durables. Therefore, the economy shifts away from nondurable expenditures. With durability, the desired demand for higher durable stock leads to accelerated durable expenditures at the cost of reducing nondurable expenditures. In particular, if the persistence of the relative price change is low, there is a higher demand to stock durables to take advantage of that lower price.

Second is the income effect due to a change in the production possibility frontier. If the price of the durable falls due to an improvement in consumer durable technology, then less resources are required to produce the same amount of durables. Therefore, the production possibility frontier of total consumption goods expands and households with higher income are willing to increase both their levels of nondurable and durable consumption.

The impulse responses suggest that the substitution effect dominates the income effect. First, in panel (a), the impact response of durables and nondurables are quantitatively stronger when the persistence of the relative price is low, due to the strong substitution effect on impact when the relative price change is expected to be temporary. However, even when the relative price is highly persistent and income effects matter, nondurable consumption falls and durable expenditures increase, implying a strong substitution channel.

Second, in panel (b), when m is high ($m = 0.90$), the economy produces mostly nondurables at the steady state. In this case, an improvement in the durable-specific technology does not significantly increase the production possibility frontier for total consumption goods. As a result, there is a small increase in hours worked. Due to the small income effect, the substitution effect from the change in the relative price dominates, which leads to a fall in nondurable expenditures. When m is low ($m = 0.60$), the economy produces more durables at the steady state and the same improvement in the durable-specific technology leads to a larger increase in hours worked. In this case, the income effect is larger and under the same utility weights on nondurable consumption, households would want to smooth out their nondurable consumption. Nevertheless, nondurables fall, because with low m, the utility weight of the household desires more consumer durables over nondurables.

Summary. In the two experiments, we find that a fall in the relative price of durables implies a strong substitution effect which generates an increase in durable expenditures and a decline in nondurable expenditures. These predictions are unique in the investment shock literature for the following reasons. In the investment shock literature, an improvement to the investment-specific technology leads to an increase in capital investment. At the same time, nondurable consumption also increases in the medium to long-run, because with more capital, the production possibility frontier has expanded, leading to a strong income effect in that frequency. However, the logic does not hold with an improvement in the consumer durable-specific technology. Household durable stocks contribute to household welfare, but not to the production of nondurable goods. As a result, nondurables and the relative price of durables move in the same direction conditional on a shock to the durable-specific technology.

3 Price and quantity of consumer durables

In this section, we empirically investigate the business-cycle dynamics of prices and quantity of consumer durables. Using the detailed Personal Consumption Expenditures (PCE) data, we document the correlation between price and quantity for various consumption goods.

In PCE, consumption expenditures are broadly classified as durable goods, nondurable goods, and services. For durable goods, we use the finest categories available in the data. There are in total 43 types of durable expenditures: 9 in Motor vehicles and parts, 10 in Furnishing and durable household equipment, 17 in Recreational goods and vehicles, and 7 in Other durable goods.

For three durable expenditures (Video discs, tapes, and permanent digital downloads; Personal computers/tablets and peripheral equipment; Computer software and accessories), the data are recorded from 1977. For two durable expenditures (Net transactions in used trucks; Used truck margin), the data start from 1983. For all the other 38 durable expenditures, the data are available from 1959.

The real quantity of each durable category is constructed by dividing its value with its price level. The relative price of each durable is constructed by dividing its price level with the price level of nondurables and services.

Table 1: Price-quantity correlation for investment goods

	10p	25p	50p	75p	90p
		Consumer durables (43)			
Growth rate:	−0.718	−0.622	−0.433	−0.266	−0.135
HP filter:	−0.740	−0.589	−0.375	−0.230	−0.083
		Equipment investment (31)			
Growth rate:	−0.434	−0.237	−0.125	−0.032	+0.063
HP filter:	−0.567	−0.290	−0.171	+0.042	+0.205

Note: Correlation between the cyclical components of the relative price and the real quantity of each series. Relative price is computed by dividing the price series by the price index of nondurables and services. HP filter refers to the cyclical components the relative price and real quantity with smoothing parameter 1,600.

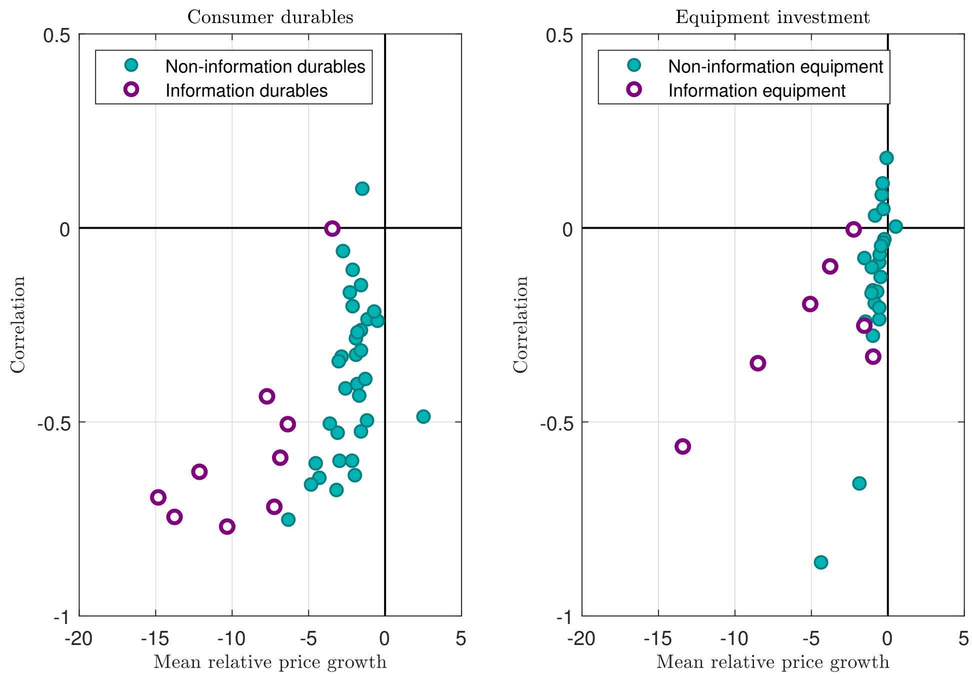
3.1 Consumer durables over the business cycle

Table 1 documents the business-cycle correlations between the relative price and real quantity for the distribution of the detailed consumer durable good, using quarterly data from 1988Q1 to 2017Q4. We also compute the same correlations for equipment investment goods in the detailed category of private investment expenditures. There are a total of 43 consumer durable goods and 31 equipment investment goods in the finest level.

We find that in terms of the price-quantity correlations, there is a lot of heterogeneity. Using growth rates, the median correlation for consumer durables is −0.433, but the 10 percentile and 90 percentile correlations are as different as −0.718 and −0.135, respectively. The same pattern holds when applying and HP filter to each series.

Another interesting feature is that the distribution of the correlation of equipment investment is more procyclical compared to that of the correlation of consumer durables. Using growth rates, the median correlation for equipment investment is −0.125. Therefore, when using the finest categories of expenditures, the negative correlation between the relative price and real

Figure 2: Investment-specific technology growth and business-cycle correlation



Note: Correlation computes the correlation between the relative price growth and the real quantity growth for each series. Each growth rate series is quarterly between 1988q1 and 2018q3. Mean relative price growth is annualized.

quantity is more apparent for consumer durables rather than equipment investment.

To understand whether the negative correlation of price and quantity is linked to innovations in technology for each investment category, we plot the average relative price growth rate against the correlation between price and quantity in Figure 2. For consumer durables, we find that goods with the biggest fall in average price growth are goods that exhibit the most apparent negative correlation between price and quantity. In particular, we classify goods into information durables and non-information durables. Information durables are a total of nine goods, which are in the categories of Video, audio, photographic, and information processing equipment, as well as Telephone and related communication equipment. Non-information durables are the remaining 34 durables.

In the left panel of Figure 2, we find that information durables are the goods that have shown the biggest fall in relative prices since 1988. At the same time, these are the goods that show the strongest negative correlation between price and quantity. Therefore, not only are information durables leading the fall in the price of durables in the long run, but their real quantites are also showing the strongest correlation with their own prices.

The right panel of Figure 2 displays the same plot for categories of equipment investment goods. There are a total of seven types of goods in the category of Information processing equipment. Price drops have been big for two goods within this category (Computers and peripheral equipment, Communication equipment), and these goods also show strong negative correlation between price and quantity. However, other information equipment investment goods are similar to non-information equipment in the distribution.

3.2 Aggregation and information processing investment

Above, we document that there is a lot of heterogeneity in terms of the correlation between price and quantity of durables. Iin this section, we look into the same correlation with aggregate variables. The first column of Table 2 displays the correlation in aggregate variables. We find that negative correlation is much less pronounced in aggregate data. The growth rate correlation is -0.221, rather than the median of -0.433 in Table 1.

We also divide aggregate durables into two types of durables: information and non-information durables. Information durables are the combination of the nine types of durables discussed above, combined by the Tornqvist method. We find that for information durables, the correlation is highly negative at -0.556. On the other hand, non-information durables are less negative at -0.201. Therefore, there is a significant difference in this moment that becomes muted when only looking at durables at the aggregate level.

Another interesting finding is that for equipment investment, the negative correlation is much more apparent at the aggregate level, which is -0.484 whereas the median for the distribution of equipment investment is -0.125.

Table 2: Price-quantity moments for aggregate investment goods

Investment good	corr(p,n)	corr(p,c)	σ_p	σ_n
		Growth rate		
Consumer durables:	−0.221	+0.001	1	1
Equipment investment:	−0.484	−0.210	1.433	1.340
Information durables:	−0.556	−0.056	1.917	0.929
Non-information durables:	−0.201	−0.049	0.989	1.105
Information equipment:	−0.472	−0.223	1.928	1.313
Non-information equipment:	−0.495	−0.150	1.641	1.686
		HP filter		
Consumer durables:	−0.049	−0.093	1	1
Equipment investment:	−0.501	−0.286	1.166	2.140
Information durables:	−0.252	−0.035	1.806	1.013
Non-information durables:	−0.063	−0.123	1.024	1.067
Information equipment:	−0.461	−0.167	1.687	1.676
Non-information equipment:	−0.574	−0.310	1.510	2.724

Note: Correlation between the cyclical components of the relative price p and the real quantity of each series, where n denotes its own quantity, and c denotes the quantity of nondurables and services. Relative price is computed by dividing the price series by the price index of nondurables and services. HP filter refers to the cyclical components the relative price and real quantity with smoothing parameter 1,600.

In this sense, complementarity across equipment investment goods appear to be much higher compared to consumer durables.

3.3 Nondurables and the relative price

In Table 2, we also report the correlation of the relative price and nondurable consumption, where nondurable consumption is defined as the aggregation of nondurable goods and services. We find that this correlation is close to zero, both for aggregate durables and even for information and non-information durables. The correlation between the relative price of equipment and nondurables is slightly negative at -0.210. Also, there is no significant difference between information and non-information equipments.

4 Full Quantitative Model

In this section, we build a quantitative business-cycles model with consumer durables. The model continues to hold the two disturbances directly linked to the dynamics of consumer durables: (i) the relative price of durables and (ii) the effective stock of durables. In addition, the model adds additional shocks to quantify real business cycles by following standard literature: productivity shock and preference shock. Finally, the model is extended to capture non-stationary movement of the relative price of investment goods in the data. We start from a model with one aggregate consumer durable, and also extend the model to allow for two consumer durables.

4.1 Households

Consider an economy populated by a large number of identical agents with preferences described over nondurable consumption C_t, stock of durables D_t, and hours worked h_t,

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(C_t, D_t, h_t),$$

where β denotes the household subjective discount factor and b_t is an exogeneous shock of preference shift.

The household sequential budget constraint is

$$C_t + P_t^d N_t + P_t^k I_t = W_t h_t + R_t u_t K_t,$$

where P_t^d and P_t^k are the relative prices of consumer durable goods and capital goods respectively. The *physical* units of durable and capital investment are denoted as N_t and I_t. Real wage is W_t and the real rental rate of existing capital is R_t, where μ_t^k measures capacity utilization in the period t. Thus $u_t K_t$ is the effective amount of physical capital in period t.

Household durable stock D_t evolves over time according to

$$D_t = (1-\delta_d)D_{t-1} + \mu_t^d N_t \left[1 - S_d\left(\frac{N_t}{N_{t-1}}\right)\right], \tag{11}$$

where $S_d(\cdot)$ is the flow adjustment cost of durables and μ_t^d measures the marginal efficiency of investment of consumer durables. The parameter δ_d governs the depreciation rate of consumer durable stock.

Capital stock K_t evolves over time according to

$$K_{t+1} = (1-\delta_k(u_t))K_t + \mu_t^k I_t \left[1 - S_k\left(\frac{I_t}{I_{t-1}}\right)\right]. \tag{12}$$

Similar to the law of motion of consumer durables, $S_k(\cdot)$ is the flow adjustment cost of capital goods and μ_t^k measures the marginal efficiency of investment of physical capital. We assume that higher capacity utilization yields faster rate of depreciation. Thus depreciation rate of the capital is an increasing and convex function of capital utilization, $\delta_k(u_t)$.

Producers of (capital and durable) investment goods are subject to linear technology:

$$I_t = z_t^k X_t^a \tilde{I}_t, \qquad N_t = z_t^d X_t^a \tilde{N}_t,$$

where $\tilde{I}_t$ and $\tilde{N}_t$ are inputs for the production of capital and durable goods, X_t^a is nonstationary investment-specific productivity, and z_t^k and z_t^n are stationary capital- and durable-specific productivity processes, respectively.

Producers of investment goods maximize the following profit functions:

$$\Pi_t^k = P_t^k I_t - \tilde{I}_t, \qquad \Pi_t^d = P_t^d N_t - \tilde{N}_t,$$

then equilibrium relative prices of capital investment good P_t^k and durable investment good P_t^d become

$$P_t^k = \frac{1}{z_t^k X_t^a}, \qquad P_t^d = \frac{1}{z_t^d X_t^a}.$$

The household's first order conditions to maximize its value with respect to C_t, D_t, h_t are

$$b_t U_1(C_t, D_t, h_t) = \Lambda_t, \tag{13}$$

$$\frac{U_3(C_t, D_t, h_t)}{U_1(C_t, D_t, h_t)} = -W_t, \tag{14}$$

$$\frac{U_2(C_t, D_t, h_t)}{U_1(C_t, D_t, h_t)} = Q_t^d - \beta \mathbb{E}_t \left[\frac{\Lambda_{t+1}}{\Lambda_t} Q_{t+1}^d \left(1 - \delta^d\right) \right], \tag{15}$$

where $\beta^t \Lambda_t$ and $\beta^t \Lambda_t Q_t^d$ are the Lagrange multipliers of the sequential budget constraint and the law of motion of consumer durables (11), respectivley. The household's first order conditions associated with K_{t+1}, u_t are

$$Q_t^k = \beta \mathbb{E}_t \left[\frac{\Lambda_{t+1}}{\Lambda_t} \left(R_{t+1} u_{t+1} + Q_{t+1}^k (1 - \delta^k(u_{t+1})) \right) \right], \tag{16}$$

$$R_t = Q_t^k \delta'(u_t), \tag{17}$$

where $\beta^t \Lambda_t Q_t$ is the Lagrange multiplier with respect to the law of motion of physical capital (12). The household's first order condition associated to N_t, I_t are

$$\frac{\Lambda_t}{z_t^k X_t^a} = Q_t^k \Lambda_t \left(1 - S^k \left(\frac{I_t}{I_{t-1}} \right) - S^{k\prime} \left(\frac{I_t}{I_{t-1}} \right) \frac{I_t}{I_{t-1}} \right) \mu_t^k + \beta \mathbb{E}_t \left[Q_{t+1}^k \Lambda_{t+1} S^{k\prime} \left(\frac{I_{t+1}}{I_t} \right) \left(\frac{I_{t+1}}{I_t} \right)^2 \mu_{t+1}^k \right], \tag{18}$$

$$\frac{z_t^k}{z_t^d} = \frac{Q_t^d \Lambda_t \left(1 - S^d\left(\frac{N_t}{N_{t-1}}\right) - S^{d\prime}\left(\frac{N_t}{N_{t-1}}\right)\frac{N_t}{N_{t-1}}\right)\mu_t^d + \beta \mathbb{E}_t\left[Q_{t+1}^d \Lambda_{t+1} S^{d\prime}\left(\frac{N_{t+1}}{N_t}\right)\left(\frac{N_{t+1}}{N_t}\right)^2 \mu_{t+1}^d\right]}{Q_t^k \Lambda_t \left(1 - S^k\left(\frac{I_t}{I_{t-1}}\right) - S^{k\prime}\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}}\right)\mu_t^k + \beta \mathbb{E}_t\left[Q_{t+1}^k \Lambda_{t+1} S^{k\prime}\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2 \mu_{t+1}^k\right]}. \tag{19}$$

4.2 Firms

Total output is produced by the effective stock of capital inputs $u_t^k K_t$ and labor inputs h_t. The production function of the firm is given by the Cobb-Douglas production function

$$\tilde{Y}_t = (u_t K_t)^\alpha (X_t h_t)^{\alpha - 1}, \tag{20}$$

where $\tilde{Y}_t$ is total production, X_t is the nonstationary labor augmented productivity process, and α is the elasticity of production with respect to effective capital.

Total output producers maximize the following profit function:

$$\Pi_t = \tilde{Y}_t - W_t h_t - R_t u_t K_t,$$

by choosing capital and labor inputs in competitive markets, the firm's demand function for capital and labor are:

$$\alpha \left(\frac{u_t K_t}{X_t h_t}\right)^{\alpha - 1} = R_t, \tag{21}$$

$$(1 - \alpha)(X_t)^{1-\alpha} \left(\frac{u_t K_t}{X_t h_t}\right)^\alpha = W_t. \tag{22}$$

4.3 Competitive Equilibrium

Total output is used either for nondurable consumption, or for inputs in the production of durable and/or capital goods. The goods market of the economy clears:

$$\tilde{Y}_t = C_t + \tilde{N}_t + \tilde{I}_t. \tag{23}$$

where

$$\tilde{N}_t = \frac{1}{z_t^d X_t^a} N_t, \tag{24}$$

$$\tilde{I}_t = \frac{1}{z_t^k X_t^a} I_t. \tag{25}$$

The exogenous processes for nondurable productivity follow the first order Markov processes:

$$\ln g_t = \rho_g \ln g_{t-1} + \sigma_g \varepsilon_{g,t}, \tag{26}$$

where $g_t \equiv X_t / X_{t-1}$. Similary, the exogenous processes for capital- and durable-specific productivity are:

$$\ln z_t^k = \rho_z^k \ln z_{t-1}^k + \sigma_z^k \varepsilon_{z,t}^k, \tag{27}$$

$$\ln z_t^d = \rho_z^d \ln z_{t-1}^d + \sigma_z^d \varepsilon_{z,t}^d, \tag{28}$$

$$\ln g_t^a = \rho_g^a \ln g_{t-1}^a + \sigma_g^a \varepsilon_{g,t}^a, \tag{29}$$

where $g_t^a \equiv X_t^a / X_{t-1}^a$. And the exogenous processes for the marginal efficiency of capital and consumer durables are:

$$\ln \mu_t^k = \rho_\mu^k \ln \mu_{t-1}^k + \sigma_\mu^k \varepsilon_{\mu,t}^k, \tag{30}$$

$$\ln \mu_t^d = \rho_\mu^d \ln \mu_{t-1}^d + \sigma_\mu^d \varepsilon_{\mu,t}^d. \tag{31}$$

Lastly, the exogenous preference shift evolves as:

$$\ln b_t = \rho_b \ln b_{t-1} + \sigma_b \varepsilon_t^b. \tag{32}$$

The innovations $\varepsilon_{g,t}, \varepsilon_{z,t}^k, \varepsilon_{z,t}^d, \varepsilon_{g,t}^a, \varepsilon_{\mu,t}^k, \varepsilon_{\mu,t}^d$ and ε_t^b are following i.i.d processes with mean zero and unitary standard deviation.

A competitive equilibrium is a set of endogeneous variables $\{C_t, D_t, h_t, P_t^d, N_t, P_t^k, I_t, W_t, R_t, u_t, K_t, \tilde{I}_t, \tilde{N}_t, \Lambda_t, Q_t^d, Q_t^k\}$ and exogeneous variables $\{g_t, z_t^k, z_t^d, g_t^a, \mu_t^k, \mu_t^d, b_t\}$ which satisfy equations (11)-(32).

4.4 Parameterization

We parameterize the utillity function U as follows:

$$U(C_t, D_t, h_t) = \frac{\left(C_t^{\gamma}\left(D_t/X_t^a\right)^{1-\gamma}(1-h_t)^{\gamma_h}\right)^{1-\sigma}-1}{1-\sigma}, \tag{33}$$

which is a Cobb-Douglas utility function of nondurables C_t, durables scaled with nonstationary investment productivity D_t/X_t^a, and leisure $1-h_t$. The reason for scaling consumer durables D_t by nonstationary investment productivity X_t^a is to ensure the stationary equilibrium, which will be discussed in section 4.5.

The depreciation function of capital utilization and adjustment cost function are parameterized as follows:

$$\delta(u) = \delta_0 + \delta_1(u-1) + \delta_2/2(u-1)^2, \tag{34}$$

$$S(x) = \frac{\kappa}{2}\left(x-\mu^k\right)^2, \tag{35}$$

by following standard parameterization in the business cycle literature.

4.5 Trend and Stationarity Transformation

The full quantitative model specifies two stochastic trends X_t and X_t^a. We model these stochastic trends to be consistent with the balanced growth path. In particular, we assume in the balanced growth path that output and investment expenditures grow at the same rate, and that effective capital goods and effective investment goods grow at the same rate.

Assuming that $F(x,y) = x^{\alpha}y^{1-\alpha}$, the balanced growth path implies that the trend for output X_t^Y is

$$X_t^Y = (X_t^a X_t^Y)^{\alpha}(X_t)^{1-\alpha}.$$

Then the trend in the effective investment good X_t^k is

$$X_t^K = X_t^a X_t^Y.$$

Based on these two relations, we get

$$\begin{aligned} X_t^Y &= X_t(X_t^a)^{\frac{\alpha}{1-\alpha}}, \\ X_t^K &= X_t(X_t^a)^{\frac{1}{1-\alpha}}. \end{aligned}$$

Since most variables in the competivie equilibrium condition are non-stationary so we should transform the variables by dividing stochastic trend. Use the relations

$$X_t^a = \left(X_t^Y\right)^{\frac{1-\alpha}{\alpha}} (X_t^z)^{\frac{\alpha-1}{\alpha}},$$

then we get the following relationships

$$\begin{aligned} \phi_t^a &= \frac{1}{\phi_t^{p_I}}, \\ \phi_t^z &= \left(\frac{(\phi_t^Y)^{\frac{1-\alpha}{\alpha}}}{\phi_t^a}\right)^{\frac{\alpha}{1-\alpha}}. \end{aligned}$$

and

$$\phi_t^k = \phi_t^a(\phi_t^Y).$$

We transform $y_t \equiv \frac{Y_t}{X_t^Y}, k_t \equiv \frac{K_t}{X_{t-1}^k}, c_t \equiv \frac{C_t}{X_t^Y}, d_t \equiv \frac{D_t}{X_t^k}, \tilde{n}_t \equiv \frac{\tilde{N}_t}{X_t^Y}, \tilde{i}_t \equiv \frac{\tilde{I}_t}{X_t^Y}, n_t \equiv \frac{N_t}{X_t^k}, i_t \equiv \frac{I_t}{X_t^k}, w_t \equiv \frac{W_t}{X_t^Y}$, $q_t^d \equiv Q_t^d X_t^a$, $q_t^k \equiv Q_t^k X_t^a, r_t \equiv R_t X_t^a, \lambda_t \equiv \Lambda_t (X_t^Y)^{\sigma}$. The full stationary competitive equilibrium conditions are in Appendix A1.

5 Quantitative Analysis

Using the quantitative model in section 4, we perform a quantitative analysis. First, we set several structural parameters based on moments of data and by following standard values from the existing literature. Second, we approximate the model by first-order using a perturbation method. Third, we estimate remaining structural parameters and other non-structural ones of the model by using Bayesian method. Finally, from the estimated and calibrated parameters, we investigate the model's prediction by examining second moments and decomposition of variables from the model.

5.1 Calibration

For steady-state worked hours $\bar{h}$ is set to 0.3 to match [...]. A preference parameter for the nondurables γ is set to be 0.6784 to match the nondurables-durables ratio $c/i_d = 3$ in the steady state. A preference parameter for the leisure γ is residually set to be 13.28 to match $\bar{h} = 0.3$. The elasticity of output to effective capital α is set to 0.37, to match the steady-state capital share in the United States. The curvature of the utility function σ is set to 2, by following standard literature. Table 3 summarizes all calibrated values. The subjective discount factor β is set to 0.99 to match annual real interest rate 4 % in the United States. Depreciation rate parameters δ_k and δ_d are both set to 0.025 by following standard literature. We set the steady state growth rate of the relative price of investment ϕ^{PI} to 0.9957 by using [...]. Similarly, we set the steady state growth rate of output ϕ^{Y} to 1.0049 by using [...]. Table 3 summarizes all calibrated parameters.

5.2 Estimation

The Bayesian estimation procedure needs prior distribution and likelihood function of the parameters to generate posterior distribution. We impose uniform prior to all parameters which reveals no prior information of the parameters. To construct likelihood function conditional on the model, we use

Table 3: Calibrated Parameters

Parameter	Value	Target
$\bar{h}$	0.3	Hours worked in the steady state
γ	0.6784	$c/i_d = 3$ in the steady state
γ_h	13.28	$h = 0.3$ in the steady state
α	0.37	Capital Share = 0.37
σ	2	CRRA curvature
β	0.99	Subjective discount factor (Q)
δ_k	0.025	Capital depreciation rate (Q)
δ_d	0.025	Durable depreciation rate (Q)
ϕ^{PI}	0.9957	Growth Rate of Rel. P of Investment
ϕ^{Y}	1.0049	Growth Rate of Output

six observables: (i) real GDP, (ii) real nondurable expenditure (including services), (iii) real durable investment, (iv) real capital investment, (v) relative price of durables over nondurable price, and (vi) relative price of capital over nondurable price. All obervations are in growth rates and the sample period is from Q1 1947 to Q3 2017. In addition, we add measurement error in real GDP growth to prevent ill-shaped likelihood function from the noise in economy-wide resource constraint (23). The likelihood evaluation is via Kalman filter iterations in the linear state space representation of the model.

The vector $\mathbf{\Theta} = [\theta, \kappa_d, \kappa_k, \delta_2, \rho_g, \rho_g^a, \rho_z^d, \rho_z^k, \rho_\mu^d, \rho_\mu^k, \rho_b, \sigma_z, \sigma_g, \sigma_z^k, \sigma_z^d, \sigma_g^a, \sigma_\mu^k, \sigma_\mu^d, \sigma_{g^y}^{ME}]$ are the set of parameters to be estimated. We construct 10 million MCMC chains using random walk Metropolis-Hastings sampler by following procedures in Herbst and Schorfheide (2016). Table 4 summarizes prior and posterior distributions from the estimation. The posterior distributions are from the last 1 million draws from the MCMC chain.

5.3 Prediction of the Model

Table 5 compares unconditional second moments from estimated model and actual data. The estimated model mathes general business cycle statistics

Table 4: Marginal Prior and Posterior Distributions for Structural Parameters

Parameter	Prior	Posterior (median and 90% ci)	
Steady-State Related Parameters			
θ	Uniform $[0, 99]$	0.92	[0.91, 0.94]
Endogenous Propagation Parameters			
κ_d	Uniform $[0, 30]$	9.95	[7.86, 12.7]
κ_k	Uniform $[0, 30]$	13.0	[10.2, 16.9]
δ_2	Uniform $[0, 10]$	5.36	[1.37, 9.55]
Orthogonal Shock Parameters			
ρ_g	Uniform $[-0.99, 0.99]$	-0.20	[-0.28, -0.11]
ρ_g^a	Uniform $[-0.99, 0.99]$	0.75	[0.67, 0.82]
ρ_z^d	Uniform $[-0.99, 0.99]$	0.98	[0.96, 0.99]
ρ_z^k	Uniform $[-0.99, 0.99]$	0.98	[0.98, 0.99]
ρ_μ^d	Uniform $[-0.99, 0.99]$	-0.47	[-0.66, -0.26]
ρ_μ^k	Uniform $[-0.99, 0.99]$	0.35	[0.21, 0.47]
ρ_b	Uniform $[-0.99, 0.99]$	0.41	[0.24, 0.60]
σ_g	Uniform $[0, 5]$	0.038	[0.032, 0.047]
σ_g^a	Uniform $[0, 5]$	0.003	[0.002, 0.004]
σ_z^d	Uniform $[0, 5]$	0.003	[0.003, 0.004]
σ_z^k	Uniform $[0, 5]$	0.005	[0.004, 0.006]
σ_μ^d	Uniform $[0, 5]$	0.06	[0.03, 0.56]
σ_μ^k	Uniform $[0, 5]$	0.42	[0.32, 0.56]
σ_b	Uniform $[0, 5]$	0.35	[0.28, 0.46]

Note: Posterior distributions are based on draws from the last 1 million draws from a 10 million MCMC chain.

well with the actual data. [...]

Table 6 shows the unconditional variance decomposition of the estimated model, we find that the estimation results suggest that the cyclical variation in durable goods are mainly explained by household preference shocks rather than investment shocks; the fall in the relative price plays a negligible role. This result is at odds with our sectoral level observation, where sectors with big price falls are sectors where prices movements and real quantity movements

Table 5: Second Moments: Data and Model

	Data	Model
Volatilities		
$\sigma\left(g^{Y}\right)$	0.86	1.38
$\sigma\left(g^{C}\right)$	0.48	0.74
$\sigma\left(g^{N}\right)$	2.97	3.08
$\sigma\left(g^{I}\right)$	3.08	3.07
$\sigma\left(g^{PD}\right)$	0.61	0.60
$\sigma\left(g^{PK}\right)$	0.76	0.72
Correlations with Output Growth		
$\rho\left(g^{C}, g^{Y}\right)$	0.55	0.53
$\rho\left(g^{N}, g^{Y}\right)$	0.59	0.78
$\rho\left(g^{I}, g^{Y}\right)$	0.61	0.86
$\rho\left(g^{PD}, g^{Y}\right)$	-0.01	0.06
$\rho\left(g^{PK}, g^{Y}\right)$	-0.21	0.09
Autocorrelations		
$\rho\left(g_t^{Y}, g_{t-1}^{Y}\right)$	0.32	0.40
$\rho\left(g_t^{C}, g_{t-1}^{C}\right)$	0.36	0.68
$\rho\left(g_t^{N}, g_{t-1}^{N}\right)$	0.00	0.16
$\rho\left(g_t^{I}, g_{t-1}^{I}\right)$	0.45	0.35
$\rho\left(g_t^{PD}, g_{t-1}^{PD}\right)$	0.53	0.47
$\rho\left(g_t^{PK}, g_{t-1}^{PK}\right)$	0.62	0.32

are highly negatively correlated. Our conjecture is that the model captures the composition shift as preference shift, as model residual. This results suggest a needs of building multiple durable goods, and exploring whether for high technology goods, the observed relative price movements can mainly account for their quantity dynamics.

6 Conclusion

Table 6: Decomposition of Variances from the Model

	g^Y	g^C	g^N	g^I	g^{PD}	g^{PK}
MEI Shock (D)	0.07	0.61	1.45	0.01	0.00	0.00
MEI Shock (K)	41.5	55.7	11.6	72.2	0.00	0.00
Nonstat. TFP Shock	1.73	3.73	1.03	0.34	0.00	0.00
Nonstat. ISP Shock	0.00	0.00	0.00	0.00	55.4	21.1
Stat. ISP Shock (D)	0.00	0.00	0.00	0.00	44.5	0.00
Stat. ISP Shock (K)	0.00	0.00	0.00	0.00	0.00	78.8
Preference Shock	56.6	40.2	85.8	27.3	0.00	0.00
Measurement error	0.00	0	0	0	0	0

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Appendix

A.1 Stationary Transformation of the Equilibrium

$$\gamma b_t \left({c_t}^{\gamma} {d_t}^{1-\gamma} - \theta \left(\frac{c_{t-1}}{\phi_t^Y} \right)^{\gamma} \left(\frac{d_{t-1}}{\phi_t^Y} \right)^{1-\gamma} \right)^{-\sigma} \cdot \left(\frac{d_t}{c_t} \right)^{1-\gamma} (1-h_t)^{\gamma_n(1-\sigma)} = \lambda_t, \tag{36}$$

$$\frac{\gamma_n}{\gamma} \left({c_t}^{\gamma} {d_t}^{1-\gamma} - \theta \left(\frac{c_{t-1}}{\phi_t^Y} \right)^{\gamma} \left(\frac{d_{t-1}}{\phi_t^Y} \right)^{1-\gamma} \right) \left(\frac{d_t}{c_t} \right)^{\gamma-1} \frac{1}{1-h_t} = w_t, \tag{37}$$

$$\frac{1-\gamma}{\gamma} \left(\frac{d_t}{c_t} \right)^{-1} = q_t^d - \beta \mathbb{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t} \left(\phi_{t+1}^Y \right)^{-\sigma} (\phi_{t+1}^a)^{-1} q_{t+1}^d (1-\delta^d) \mu_{t+1}^d \right]. \tag{38}$$

$$d_t = \left(1-\delta^d\right) \frac{d_{t-1}}{\phi_t^k} + \mu_t^d n_t \left[1 - S\left(\frac{n_t}{n_{t-1}} \phi_t^k \right) \right], \tag{39}$$

$$k_{t+1} = (1-\delta(u_t)) \frac{k_t}{\phi_t^k} + \mu_t^k i_t \left[1 - S\left(\frac{i_t}{i_{t-1}} \phi_t^k \right) \right], \tag{40}$$

$$q_t^k = \beta \mathbb{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t} \left(\phi_{t+1}^Y \right)^{-\sigma} (\phi_{t+1}^a)^{-1} \left(r_{t+1} u_{t+1} + q_{t+1}^k (1-\delta^k(u_{t+1})) \right) \right], \tag{41}$$

$$r_t = q_t \delta'(u_t) \tag{42}$$

$$\begin{aligned} \frac{1}{a_t^k} = {} & q_t^k \left(1 - S^k \left(\frac{i_t}{i_{t-1}} \phi_t^k \right) - S^{k\prime} \left(\frac{i_t}{i_{t-1}} \phi_t^k \right) \frac{i_t}{i_{t-1}} \phi_t^k \right) \mu_t^k \\ & + \beta \mathbb{E}_t \left[q_{t+1}^k \frac{\lambda_{t+1}}{\lambda_t} \left(\phi_{t+1}^Y \right)^{-\sigma} (\phi_{t+1}^a)^{-1} S^{k\prime} \left(\frac{i_{t+1}}{i_t} \phi_{t+1}^k \right) \left(\frac{i_{t+1}}{i_t} \phi_{t+1}^k \right)^2 \mu_{t+1}^{\delta,k} \right], \end{aligned} \tag{43}$$

$$\frac{a_t^k}{a_t^d} = \frac{q_t^d \left(1 - S^d \left(\frac{n_t}{n_{t-1}} \phi_t^k \right) - S^{d\prime} \left(\frac{n_t}{n_{t-1}} \phi_t^k \right) \frac{n_t}{n_{t-1}} \phi_t^k \right) \mu_t^d + \beta \mathbb{E}_t \left[q_{t+1}^d \frac{\lambda_{t+1}}{\lambda_t} \left(\phi_{t+1}^Y \right)^{-\sigma} (\phi_{t+1}^a)^{-1} S^{d\prime} \left(\frac{n_{t+1}}{n_t} \phi_{t+1}^k \right) \left(\frac{n_{t+1}}{n_t} \phi_{t+1}^k \right)^2 \mu_{t+1}^d \right]}{q_t^k \left(1 - S^k \left(\frac{i_t}{i_{t-1}} \phi_t^k \right) - S^{k\prime} \left(\frac{i_t}{i_{t-1}} \phi_t^k \right) \frac{i_t}{i_{t-1}} \phi_t^k \right) \mu_t^{\delta,k} + \beta \mathbb{E}_t \left[q_{t+1}^k \frac{\lambda_{t+1}}{\lambda_t} \left(\phi_{t+1}^Y \right)^{-\sigma} (\phi_{t+1}^a)^{-1} S^{k\prime} \left(\frac{i_{t+1}}{i_t} \phi_{t+1}^k \right) \left(\frac{i_{t+1}}{i_t} \phi_{t+1}^k \right)^2 \mu_{t+1}^{\delta,k} \right]}, \tag{44}$$

$$\alpha \left(u_t \frac{k_t}{\phi_t^k h_t} \right)^{\alpha-1} = r_t, \tag{45}$$

$$(1-\alpha)\left(u_t \frac{k_t}{\phi_t^k h_t}\right)^{\alpha} = w_t, \tag{46}$$

$$\tilde{y}_t = \left(u_t \frac{k_t}{\phi_t^k}\right)^{\alpha} h_t^{1-\alpha}, \tag{47}$$

$$\tilde{n}_t = \frac{1}{z_t^d} n_t, \tag{48}$$

$$\tilde{i}_t = \frac{1}{z_t^k} i_t, \tag{49}$$

$$\tilde{y}_t = c_t + \tilde{n}_t + \tilde{i}_t, \tag{50}$$

$$\phi_t^Y = \phi_t^z \phi_t^{a\,\frac{\alpha}{1-\alpha}}, \tag{51}$$

$$\phi_t^k = \phi_t^a \phi_t^Y, \tag{52}$$

A2. Steady State Solutions

In the steady state, $u = 1$, $\varepsilon^{\delta,d} = \varepsilon^{\delta,k} = 1$, and calibrated parameters,

$$\phi^a = \frac{1}{\phi^{P_I}},$$

$$\phi^z = \left(\frac{\phi^{Y\,\frac{1-\alpha}{\alpha}}}{\phi^a}\right)^{\frac{\alpha}{1-\alpha}},$$

$$\phi^k = \phi^a \phi^Y,$$

$$\delta_1 = \frac{\phi^a}{\beta \phi^{Y^{-\sigma}}} + \delta_0 - 1$$

$$q^k = 1,$$

$$r = q^k \delta_1,$$

$$k = \phi^k h \left(\frac{r}{\alpha}\right)^{\frac{1}{\alpha-1}},$$

$$w = (1-\alpha)\left(\frac{k}{\phi^k h}\right)^{\alpha},$$

$$i = \left(1 - \frac{1-\delta_0}{\phi^k}\right) k,$$

$$\tilde{i} = i,$$

$$q^d = q^k,$$

$$\frac{c}{d} = \left[\frac{q^d \gamma}{1-\gamma}\left(1 - \beta \phi^{Y-\sigma} \phi^{a-1}(1-\delta^d)\right)\right],$$

$$y = \left(\frac{k}{\phi^k}\right)^{\alpha} h^{1-\alpha},$$

$$d = \frac{y - i_k}{\frac{c}{d} + \left(1 - \frac{1-\delta^d}{\phi^k}\right)},$$

$$n = \left(1 - \frac{1-\delta^d}{\phi^k}\right) d,$$

$$\tilde{n} = n,$$

$$c = \frac{c}{d} \cdot d,$$

$$\gamma_n = \frac{\gamma w (1-h)\left(\frac{d}{c}\right)^{1-\gamma}}{\left(c^{\gamma} d^{1-\gamma} - \theta\left(\frac{c}{\phi^Y}\right)^{\gamma}\left(\frac{d}{\phi^Y}\right)^{1-\gamma}\right)},$$

$$\lambda = \gamma\left(c^{\gamma} d^{1-\gamma} - \theta\left(\frac{c}{\phi^Y}\right)^{\gamma}\left(\frac{d}{\phi^Y}\right)^{1-\gamma}\right)^{-\sigma} \cdot \left(\frac{d}{c}\right)^{1-\gamma}(1-h)^{\gamma_n(1-\sigma)}.$$